Midterm, MAT334 - Complex Variables, June 30, 2011

Last Name: ____________________ Name: ____________________ Student No.: ____________________

This test has **five** questions. Simplify your answers as much as possible (but not more than that).

1. (*20 points*) a) Find all complex numbers z satisfying

$$(2/\bar{z} + 1)^3 = 1$$

Write the answers in the form $a + ib$

b) Sketch the set of complex numbers satisfying $|2/\bar{z} + 1|^3 = 1$

Is this set open?

Closed?

Connected?

Does it contain points not in its boundary?

2. (*20 points*) a) Find the radius of convergence of

$$\sum_{n=0}^{\infty} e^{(1+i)n} z^n$$

b) Compute

$$\sum_{n=1}^{\infty} n \left(\frac{1+2i}{5} \right)^{n-1}$$

or show that the sum diverges.

3. (*20 points*) a) Evaluate the integral

$$\int_{|z-3i|=2} \frac{e^{\pi z/2}}{z^2+4} dz$$

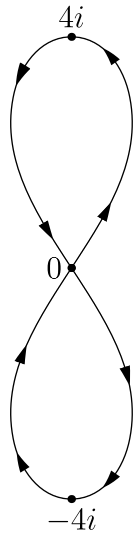
where the integration is along the circle $|z-3i|=2$ traversed counter-clockwise.
b) Evaluate the integral

$$\int_{|z+3i|=2} \frac{e^{\pi z/2}}{z^2+4} dz$$

where the integration is along the circle $|z+3i|=2$ traversed counter-clockwise.
c) Evaluate the integral

$$\int_{C} \frac{e^{\pi z/2}}{z^2+4} dz$$

where C is the curve in the following picture:



4. (*20 points*) a) Let

$$f(z) = \begin{cases} \frac{|z|^2-1}{z-1} & \text{if } z \neq 1 \\ 2 & \text{if } z = 1 \end{cases}$$

Determine whether the function $f(z)$ is continuous at the point $z = 1$ (explain why or why not).

b) Let $g(z) = z^2 - |z|^2$. Determine all the points where $g'(z)$ exists. Find $g'(z)$ at each such point.

5. (*20 points*) Suppose that the function f is analytic (i.e. complex-differentiable) at all points of the complex plane. Suppose also that the image of f is contained in the unit circle $\{u + iv | u^2 + v^2 = 1\}$. Show that f is constant.